

# 2021 Academic Scholarship Examination Paper STAGE TWO 

MATHEMATICS II

23 - 25 February 2021
Time allowed: 1 hour

## PLEASE WRITE IN BLACK INK

Please write your answers on separate sheets of paper, using only one side of each sheet of paper that you use.

> Show all working.

Calculators can be used.

1. a. A regular hexagon, $A B C D E F$, is drawn inside a circle of centre $O$ and radius $r$, as in the diagram below. Find an expression, in term of $r$, for the area of the hexagon.

b. A smaller circle is now drawn inside the same hexagon, as in the diagram below. Find and expression, in terms of $r$, for the area of the smaller circle.

2. Modelling the pandemic, a scientist claims that the number of cases is decreasing by $30 \%$ each week. In week 6 there are 30000 cases.
a. Calculate the number of cases in week 7.
b. Calculate the number of cases in week 5 .
c. Calculate the week in which the number of cases will first fall below 1000.
3. a. Solve the following simultaneous equations:

$$
\left.\begin{array}{l}
3 x+4 y=43 \\
2 x-5 y=-2
\end{array}\right\}
$$

b. Use your answers to part a. to solve the following simultaneous equations:
i. $\left.\quad \begin{array}{rl}0.3 x+0.4 y & =43 \\ 0.2 x-0.5 y & =-2\end{array}\right\}$
ii. $\left.\begin{array}{rl}3 x^{2}+4 y^{2} & =43 \\ 2 x^{2}-5 y^{2} & =-2\end{array}\right\}$
iii. $\left.\begin{array}{c}3 x+4 y=43 x y \\ 2 x-5 y=-2 x y\end{array}\right\}$
4. Peter is going on a 60 km cycle ride. He splits his ride into three stages of equal distance.

- On the first stage he cycles at a speed of $v \mathrm{~km} / \mathrm{h}$
- On the second stage he increases his speed by $2 \mathrm{~km} / \mathrm{h}$
- On the final stage he cycles twice as fast as on the first stage

The whole ride takes 5 hours and 45 minutes.
Write down an equation for $v$, and solve it to find his speed for the final stage.
[Hint: $8 \times 30=240$ ]
5. Write down any five consecutive numbers.
a. Add the first number to the last number, and divide your answer by two. What do you get? Use algebra to explain it.
b. Multiply the first number by the last number, and add four to your answer. What do you get? Use algebra to explain it.
c. Square each of the numbers, and add the five squares together. Then subtract ten from your answer, and then divide that answer by five. What do you get? Use algebra to explain it.
d. Add the cube of the first number to cube of the last number. Then subtract 24 times the middle number from your answer, and then divide that answer by two. What do you get? Use algebra to explain it.
6. a. ABC is an equilateral triangle. Angles are as shown on the diagram below. Use algebra to find the values of $x, y$ and $z$.

b. $P Q R$ is an isosceles triangle in which $P Q=P R$. Angles are as shown on the diagram below. Given that the largest angle is $102^{\circ}$, use algebra to find the values of $x, y$ and $z$.


